ECE 215 Spring 2025

Objective 1.3:

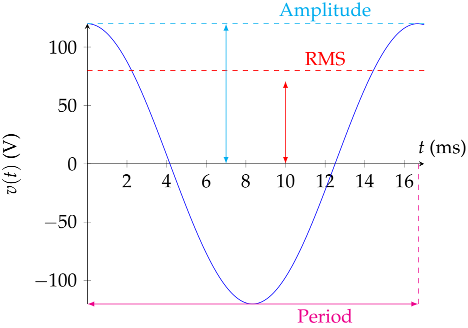
AC Circuit Analysis

UNITED STATES
AIR FORCE
ACADEMY

Objective 1.3

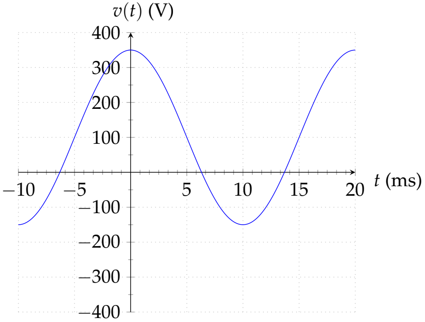
I can calculate the voltages, currents, instantaneous power, and average power associated with devices in a simple AC-powered circuit using tools such as KVL, KCL, voltage and current dividers, Ohm’s Law, RMS values, and the power equation.

Anatomy of AC



- $v(t) = \mathrm{V_{bias}} + \mathrm{A}\cos 2\pi f \cdot t + \Phi$
- $\mathrm{V_{bias}}$ = DC bias
- Amplitude (A or $\mathrm{V_{pk}}$) = peak strength of the signal
- 2π = converts period to $^\circ$(sometimes 360°)
- f = frequency $\left(Hz = \frac{1}{s}\right)$
- t = time
- Period (T) = the time range over which the signal repeats $\left(T = \frac{1}{f}\right)$
- Φ = Phase shift $\left(\Phi = \frac{2\pi}{\mathrm{T}}\Delta t\right)$
- RMS Value ($\mathrm{V_{rms}}$) = effective value of the signal

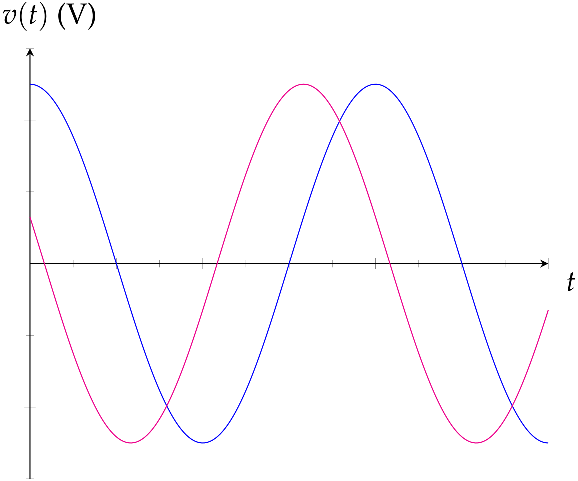
AC Example - Sine Signal



Questions:

- What is DC bias?
- What is the phase shift?
- What is the equation for this graph?
- What is the value of the signal at $t = 10ms$?
- What is the value of the signal at $t = 0ms$?

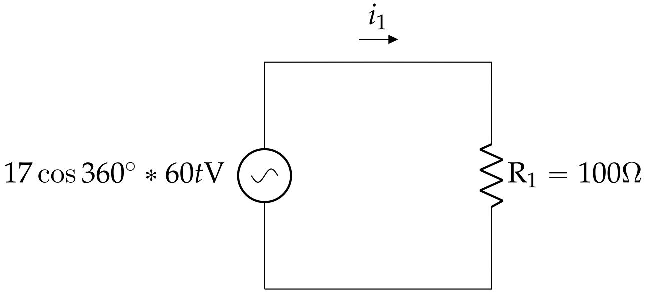
Phase Angle



Red leads Blue by 75 deg, or alternatively Blue lags Red by 75 deg.

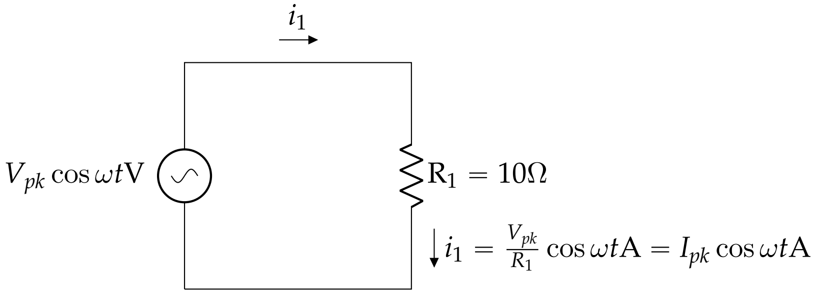
AC and Ohm's Law - Good News

- Ohm's Law still holds true!
- Must keep cosine term (still AC)
- Find i_1



AC and Ohm's Law - Good News

- When looking at just a resistor, there is no phase difference between V_s and I_s
- No phase difference → they are in phase!

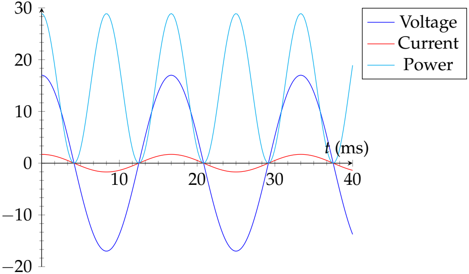


- Recall $\omega = 2\pi f$

What About Power?

Question: how much power is consumed by the resistor?

$$v(t) = 17\cos 450t \text{V} \qquad i(t) = 1.7\cos 450t \text{A}$$



Average value of $\mathrm{P_{inst}}(t)$?

Instantaneous Power

$$\mathrm{P_{inst}}(t) = \mathrm{V_R I_R} =$$

RMS

Definition

RMS value for a periodic waveform is the equivalent DC value that produces the same power effect.

$$\mathrm{V_{RMS}} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \left[\mathrm{V_{pk}}\cos\omega t + \theta\right]^2 dt}$$
$$= \frac{\mathrm{V_{pk}}}{\sqrt{2}} = 0.7071\mathrm{V_{pk}}$$

Real Power

RMS

$$V_{RMS} = \sqrt{V_{bias}^2 + \frac{A^2}{2}} \qquad \text{if } V_{bias} = 0\text{, then:}$$

$$V_{RMS} = \frac{V_{pk}}{\sqrt{2}} = 0.707 * V_{pk}$$

Real Power

The average of the instantanous power; represents the real work done by the electrical system.

$$P = \frac{1}{2} V_{pk} I_{pk} = \frac{1}{2} \left(\qquad V_{rms} \right) \left(\qquad I_{rms} \right)$$