Practice Problems

1. Convert and plot all of these values on the complex plane
    a. Convert to rectangular:
        i. $10\angle{-10^0}$
        $10*\cos(-10^0) + j10*\sin(-10^0) = 9.85 – j1.74$

        ii. $5\angle 160^0$
        $5*\cos(160^0) + j5*\sin(160^0) = -4.7 + j1.71$

        iii. $7\angle 50^0$
        $7*\cos(50^0) + j7*\sin(50^0) = 4.5 + j5.36$

    b. Convert to phasor
        i. - 10 + j6

A = $\sqrt{10^2 + 6^2} = 11.66;\ \theta = \tan^{-1}\left(\frac{6}{-10}\right) = 149^0$

Answer: $11.66\angle 149^0$

        ii. 5 - j6

A = $\sqrt{5^2 + 6^2} = 7.81;\ \theta = \tan^{-1}\left(\frac{-6}{5}\right) = -50.2^0$

Answer: $7.81\angle{-50.2^0}$

        iii. -17 – j4

A = $\sqrt{17^2 + 4^2} = 17.5;\ \theta = \tan^{-1}\left(\frac{-4}{17}\right) = 193.2^0$

Answer: $17.5\angle 193.2^0$

2. Perform the requested mathematical operation:
   1. (10+j5)+(-4-j16)

      6-j11

   2. (10+j5)*(-4-j16)

      40-j180

   3. $\frac{12\angle 15^0}{3\angle 27^0}$

      $4\angle -12^0$

   4. $12\angle 150^0 + 6\angle 27^0$

      $10\angle 120^0$

   5. $5\angle 57^0 * 6\angle 36^0$

      $30\angle 93^0$

3. Perform the following computations, preferably by hand. (Use your calculator to check your answers.)
   a. (j6)(j3) = j^2*18 = -18

   b. j5 + 5 + j8 + 7 = (7 + 5) + j(5 + 8) = 12 + j13

   c. $\frac{j18}{j3}$ = 6

   d. (8 + j6)(j8 + 2) = j64 + 16 + $j^2$48 + j12 = -32 + j76

   e. $\frac{1}{8+j5} + \frac{1}{8+j7} = \frac{8+j7+8+j5}{(8+j7)*(8+j5)} = \frac{16+j12}{64+j40+j56+(-35)} = \frac{(16+j12)(29-j96)}{(29+j96)(29-j96)} = \frac{(1616-j1188)}{(10057)} =$
   $0.1607 - j0.118$

4. Convert your answers from Problem 3 above to phasor notation
   a. (j6)(j3) = j^2*18 = -18 = $18\angle 0^0$

   b. j5 + 5 + j8 + 7 = (7 + 5) + j(5 + 8) = 12 + j13

   A = $\sqrt{12^2 + 13^2} = 17.7;\ \theta = \tan^{-1}\left(\frac{13}{12}\right) = 47.3^0$

   Answer: $17.7\angle 47.3^0$

   c. $\frac{j18}{j3}$ = 6 = $6\angle 0^0$

   d. (8 + j6)(j8 + 2) = j64 + 16 + $j^2$48 + j12 = -32 + j76

   A = $\sqrt{32^2 + 76^2} = 82.46;\ \theta = \tan^{-1}\left(\frac{76}{-32}\right) = 67.17^0$

   Answer: $82.46\angle 67.17^0$

   e. $\frac{1}{8+j5} + \frac{1}{8+j7} = \frac{8+j7+8+j5}{(8+j7)*(8+j5)} = \frac{16+j12}{64+j40+j56+(-35)} = \frac{(16+j12)(29-j96)}{(29+j96)(29-j96)} = \frac{(1616-j1188)}{(10057)} =$
   $0.1607 - j0.118$

   A = $\sqrt{0.1607^2 + 0.118^2} = 0.1994;\ \theta = \tan^{-1}\left(\frac{-0.118}{0.1607}\right) = -36.29^0$

   Answer: $0.1994\angle -36.29^0$

5. Convert the following from phasor notation to rectangular coordinates:
   a. $6\angle 30^0 = 6\cos(30^0) + j6\sin(30^0) = 5.196 + j3$

   b. $2\angle 90^0 = 2\cos(90^0) + j2\sin(90^0) = j2$

   c. $18\angle 45^0 = 18\cos(45^0) + j18\sin(45^0) = 12.73 + j12.73$

   d. $7\angle 54^0 = 7\cos(54^0) + j7\sin(54^0) = 4.11 + j5.66$

6. Perform the following calculations:
   a. $(6\angle 30^0)(2\angle 90^0) = (6*2)\angle(30^0 + 90^0) = 12\angle 120^0$

   b. $\frac{2\angle 90^0}{6\angle 30^0} = \frac{2}{6}\angle(90^0 - 30^0) = 0.333\angle 60^0$

   c. $6\angle 30^0 + 2\angle 90^0 = 5.196 + j3 + j2 = 5.196 + j5$

   A = $\sqrt{5.196^2 + 5^2} = 7.21;\ \theta = \tan^{-1}\left(\frac{5}{5.196}\right) = 43.9^0$

   Answer: $7.21\angle 43.9^0$

   d. $2\angle 90^0 - 6\angle 30^0 = j2 - (5.196 + j3) = -5.196 - j1$

   A = $\sqrt{5.196^2 + 1^2} = 5.29;\ \theta = \tan^{-1}\left(\frac{-1}{-5.196}\right) = 190.9^0$

   Answer: $5.29\angle 190.9^0$ – NOTE: The negative signs must be observed with regards to the angle. The double negative states that the angle lies in the Third Quadrant, however, you calculator will give you an answer in the First Quadrant. You must make sure to add 180^0 to get the correct angle.

   e. $\frac{6\angle 30^0}{3\angle 90^0} = \frac{6}{3}\angle(30^0 - 90^0) = 2\angle -60^0$

7. Convert the following values into impedances:
   a. C = 10µF, f = 200 Hz

$$Z_c = \frac{-j}{2\pi * 200Hz * (10 * 10^{-6}F)} = -j79.6\Omega$$

   b. L = 20 mH, f = 20 Hz
      $Z_L = j\omega L = j*2\pi*20Hz*0.020H = j2.51\Omega$

   c. R = 15Ω, f = 100 Hz
      $Z_R = R = 15\Omega$

8. In a given circuit, if $v_s(t) = 100\cos(360^0 1kt)$V, determine the impedances of the following components:
   a. R = 100 Ω
      $Z_R = R = 100\Omega$

   b. C = 66 µF

$$Z_c = \frac{-j}{2\pi * 1000Hz * (66 * 10^{-6}F)} = -j2.41\Omega$$

   c. L = 10 mH
      $Z_L = j\omega L = j*2\pi*1000Hz*0.010H = j62.8\Omega$
9. In a given circuit, if $v_s(t) = 100\cos(360^0 500t)$V, determine the impedances of the following components:
   a. R = 150 Ω
      $Z_R = R = 150\Omega$

   b. C = 270 µF

$$Z_c = \frac{-j}{2\pi * 500Hz * (270 * 10^{-6}F)} = -j1.18\Omega$$

   c. L = 144 mH
      $Z_L = j\omega L = j*2\pi*500Hz*0.144H = j452\Omega$

10. Convert the voltage source values to phasor RMS values
    a. $v_s(t) = 5\cos(360° * 100 * t)\,V$

       The voltage source transforms to an RMS phasor by:
       $\tilde{V}_s = \frac{5}{\sqrt{2}}\angle 0^0 = 3.54V_{RMS}\angle 0^0$

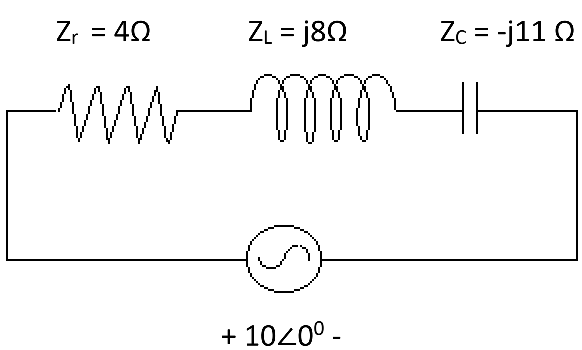
    b. $v_s(t) = 377\cos(360° * 60k * t + 30°)\,V$

       The voltage source transforms to an RMS phasor by:
       $\tilde{V}_s = \frac{377}{\sqrt{2}}\angle 30^0 = 266.54V_{RMS}\angle 30^0$

    c. $v_s(t) = 169.73\cos(360° * 400 * t - 45°)\,V$

       The voltage source transforms to an RMS phasor by:
       $\tilde{V}_s = \frac{169.73}{\sqrt{2}}\angle -45^0 = 120V_{RMS}\angle -45^0$

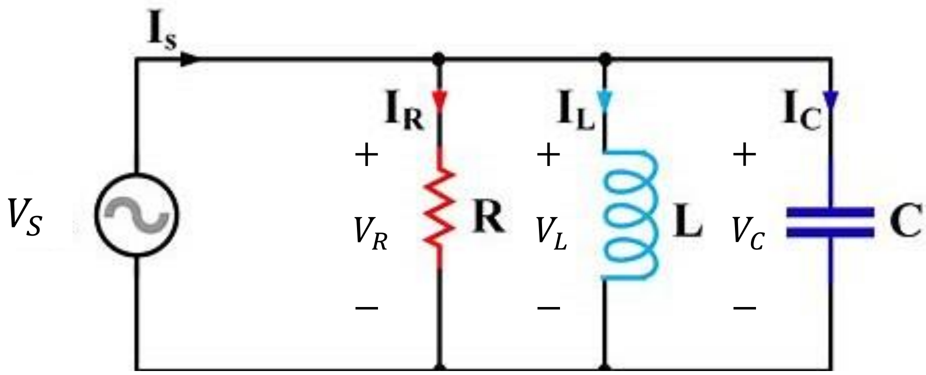
11. For the circuit below, determine the equivalent impedance. Then write a voltage divider equation to determine the voltage drop over the inductor.



$Z_{eq} = Z_r + Z_L + Z_C = 4 + j8 + -j11 = (4 - j3)\Omega = 5\angle\text{-}36.9^0$

$V_L = \frac{Z_L}{Z_{eq}}V_{in} = \frac{j8}{4-j3}(10\angle 0°) = 16\angle 126.9^0$

12. The circuit below is operating at 400 Hz. Determine the equivalent impedance. What is the voltage and current drop over each component when $V_S = 150V\angle 0°$, $R = 1k\Omega$, $L = 30\ mH$, and $C = 20\ \mu F$? Also, find I_S without using Z_{eq}



Z_R = 1kΩ Z_L = jωL = j*2*π*400Hz*0.03H = j75.4Ω $Z_C = \frac{-j}{2\pi * 400Hz * 20x10^{-6}F} = -j19.9\Omega$

$$Z_{eq} = \left(\frac{1}{1000} + \frac{1}{j75.4} + \frac{1}{-j19.9}\right)^{-1} = 729.8m\Omega - j27\Omega = 27\Omega\angle -88^0$$

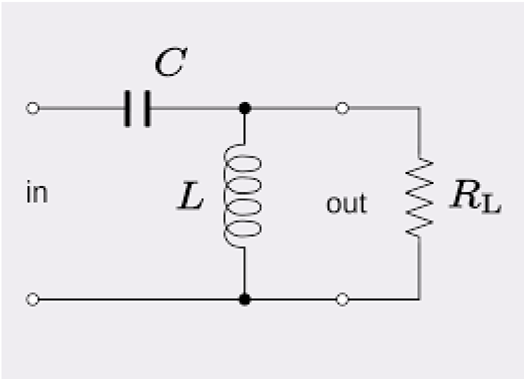
$V_s = V_R = V_L = V_C = 150V\angle 0^0$

$$I_R = \frac{V_R}{Z_R} = \frac{150V\angle 0^0}{1000\Omega} = 150mA\angle 0^0 \qquad I_L = \frac{V_L}{Z_L} = \frac{150V\angle 0^0}{j75.4\Omega} = 1.99A\angle -90^0$$

$$I_C = \frac{V_C}{Z_C} = \frac{150V\angle 0^0}{-j19.9\Omega} = 7.54A\angle 90^0$$

By KCL, $I_S = I_R + I_L + I_C = 150mA\angle 0^0 + 1.99A\angle -90^0 + 7.54A\angle 90^0 = 5.55A\angle 88.5°$

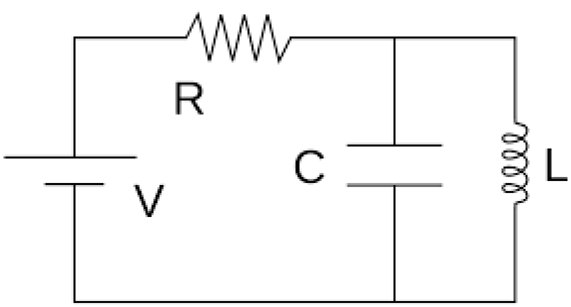
13. For the circuit below, determine the equivalent impedance, given the input frequency is 2kHz, L = 27mH, C = 150nF, and R = 5kΩ



Z_R = R = 5kΩ Z_L = jωL = j*2*π*2000Hz*0.027H = j339.3Ω $Z_C = \frac{-j}{2\pi * 2000Hz * 150x10^{-9}F} = -j530.5\Omega$

$$Z_{eq} = -j530.5\Omega + \left(\frac{1}{5000\Omega} + \frac{1}{j339.3\Omega}\right)^{-1} = 22.9 - j193\Omega = 194\Omega\angle - 83.2°$$

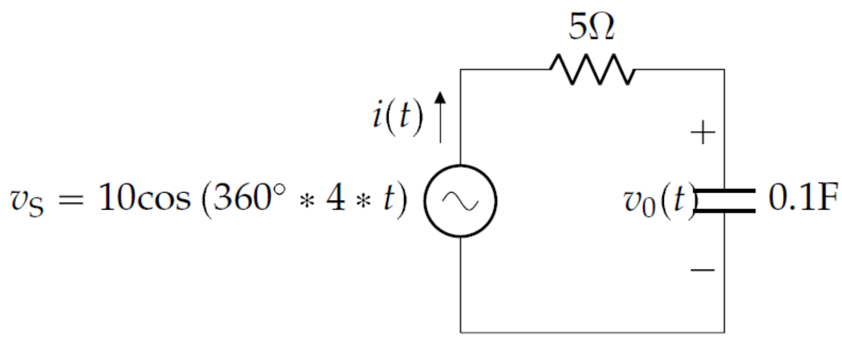
14. For the circuit below, determine the equivalent impedance, given the input frequency is 60Hz, R = 20 Ω, C = 15nF, and L = 2mH. (Assume the voltage source is AC despite the DC source symbol.)



Z_R = 20Ω Z_L = jωL = j*2*π*60Hz*0.002H = j754.0mΩ $Z_C = \frac{-j}{2\pi * 60Hz * 15x10^{-9}F} = -j176.8k\Omega$

$$Z_{eq} = 20 + \left(\frac{1}{-j176800} + \frac{1}{j0.7540}\right)^{-1} = 20 + j0.754\Omega = 20.0\Omega\angle 2.16°$$

15. For the circuit below, find $v_0(t)$ and i(t)



$Z_R = 5\Omega$ $\qquad Z_C = \frac{-j}{2\pi*4Hz*0.1F} = -j398m\Omega$ $\qquad Z_{eq} = 5\Omega - j\,398m\Omega$

$$\widetilde{V_s} = \frac{10}{\sqrt{2}} V_{RMS}\angle 0° = 7.07 V_{RMS}\angle 0°$$

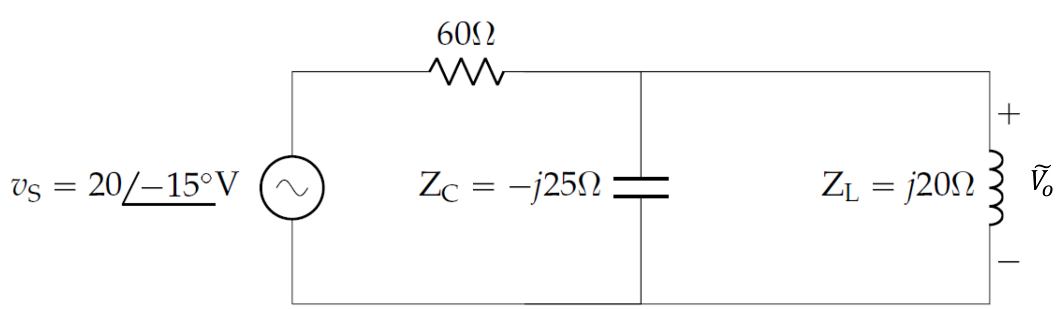
$$\widetilde{V_o} = \frac{Z_C}{Z_{eq}}\widetilde{V_s} = \frac{-j0.398}{5-j0.398}(7.07V_{RMS}\angle 0°) = (0.0793\angle-85.4°)(7.07V_{RMS}\angle 0°) = 561mV\angle{-85.4^0}$$

$$v_o(t) = (561)*(\sqrt{2})\cos(360°*4*t-85.4°)\,mV = 793\cos(360°*4*t-85.4°)mV$$

$$I_S = \frac{\widetilde{V_s}}{Z_{eq}} = \frac{7.07V_{RMS}\angle 0^0}{5-j0.398} = \frac{7.07V_{RMS}\angle 0°}{5.016\Omega\angle -4.55°} = 1.41A_{RMS}\angle 4.55^0$$

$$i_s(t) = (1.41)(\sqrt{2})cos(360°*4*t+4.55°)A = 1.99cos(360°*4*t+4.55°)A$$

16. For the circuit below, find $\widetilde{V_o}$ and the current flowing through the capacitor



$Z_R = 60\Omega$ $\qquad Z_C = -j25\Omega$ $\qquad Z_L = j20\Omega$

$$Z_{eq} = Z_R + Z_{CL} = 60 + \left(\frac{1}{-j25}+\frac{1}{j20}\right)^{-1} = 60\Omega + j100\Omega = 116.6\Omega\angle 59.04^0$$

$$\widetilde{V_o} = \frac{Z_{CL}}{Z_{eq}} V_S = (j100)/(60+j100)*20V\angle\text{-}15° = (0.857\angle 31°)20V\angle\text{-}15° = 17.1V\angle 16°$$

$$I_C = \frac{V_0}{Z_C} = \frac{17.1V\angle 16°}{-j25} \quad = \frac{17.1V\angle 16°}{25\Omega\angle -90°} = 684mA\angle 106°$$