Practice Problems

1. Calculate the cutoff frequency for a circuit with:
   a. R = 1kΩ and C = 3nF

$$f_{cutoff} = \frac{1}{2\pi RC} = \frac{1}{2\pi(1000)(3x10^{-9})} = 53kHz$$

   b. R = 5kΩ and C = 1.5μF

$$f_{cutoff} = \frac{1}{2\pi RC} = \frac{1}{2\pi(5000)(1.5x10^{-6})} = 21.22Hz$$

2. What should the resistor value be for the following filters:
   a. f_{cutoff} = 1.5kHz and C = 500 nF

$$f_{cutoff} = \frac{1}{2\pi RC} - this\ means\ R = \frac{1}{2\pi f_{cutoff} C} = \frac{1}{2\pi(1500)(500x10^{-9})} = 212\Omega$$

   b. f_{cutoff} = 417Hz and C = 56nF

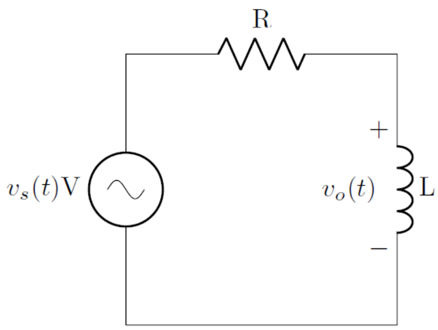
$$f_{cutoff} = \frac{1}{2\pi RC} - this\ means\ R = \frac{1}{2\pi f_{cutoff} C} = \frac{1}{2\pi(417)(56x10^{-9})} = 6.8k\Omega$$

   c. f_{cutoff} = 2kHz and C = 500 μF

$$f_{cutoff} = \frac{1}{2\pi RC} - this\ means\ R = \frac{1}{2\pi f_{cutoff} C} = \frac{1}{2\pi(2000)(500x10^{-6})} = 159m\Omega$$

3. Are the circuits below high or low pass filters? How do you know?

(a)



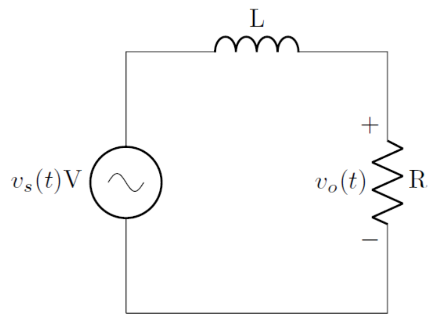
$$\frac{v_0(t)}{v_s(t)} = \frac{1}{1 + \frac{R}{j2\pi fL}}$$

when f = 0, $\frac{v_0(t)}{v_s(t)}$ = 0 - When the frequency is low, the signal is not passed.

when f = ∞, $\frac{v_0(t)}{v_s(t)}$ = 1 - When the frequency is high, the signal is passed.

High Pass Filter!

(b)



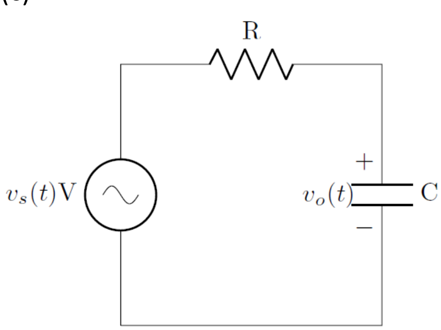
$$\frac{v_0(t)}{v_s(t)} = \frac{R}{j2\pi fL + R}$$

when f = 0, $\frac{v_0(t)}{v_s(t)}$ = 1 - When the frequency is low, the signal is passed.

when f = ∞, $\frac{v_0(t)}{v_s(t)}$ = 0 - When the frequency is high, the signal is not passed.

Low Pass Filter!

(c)



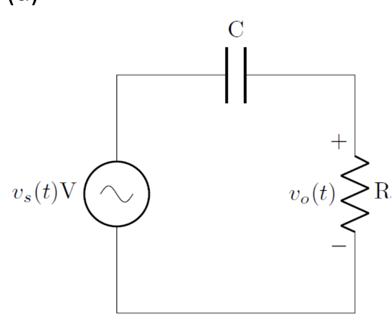
$$\frac{v_0(t)}{v_s(t)} = \frac{1}{j2\pi fCR + 1}$$

when f = 0, $\frac{v_0(t)}{v_s(t)}$ = 1 - When the frequency is low, the signal is passed.

when f = ∞, $\frac{v_0(t)}{v_s(t)}$ = 0 - When the frequency is high, the signal is not passed.

Low Pass Filter!

(d)



$$\frac{v_0(t)}{v_s(t)} = \frac{1}{1 + \frac{1}{j2\pi fRC}}$$

when f = 0, $\frac{v_0(t)}{v_s(t)}$ = 0 - When the frequency is low, the signal is not passed.

when f = ∞, $\frac{v_0(t)}{v_s(t)}$ = 1 - When the frequency is high, the signal is passed.

High Pass filter!

4. Calculate the cutoff frequency of the following systems.

(a) A transmission line modeled as an R-L circuit with R=4 Ω and L=5µH.

$$f_{cutoff} = \frac{R}{2\pi L} = \frac{4}{2\pi(5x10^{-6})} = 127kHz$$

(b) An R-C low pass filter with R=60 Ω and C=5nF

$$f_{cutoff} = \frac{1}{2\pi RC} = \frac{1}{2\pi(60)(5x10^{-9})} = 531kHz$$

(c) A C-R high pass filter with R=100 Ω and C=8µF.

$$f_{cutoff} = \frac{1}{2\pi RC} = \frac{1}{2\pi(100)(8x10^{-6})} = 199Hz$$

5. Your communications radio has a lower frequency bound of 800kHz. You know it has a capacitor value of 100nF, but what is the resistor value?

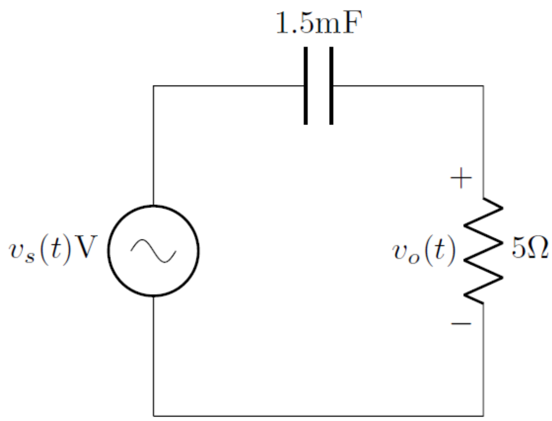
$$f_{cutoff} = \frac{1}{2\pi RC} - this\ means\ R = \frac{1}{2\pi f_{cutoff} C} = \frac{1}{2\pi(800000)(100x10^{-9})} = 1.99\Omega$$

6. Design a high pass filter to get rid of a DC bias (0Hz) using a 100-Ω resistor you have available.

DC bias means f = 0Hz. This is impossible to attain, so let's choose our cutoff frequency to be 10Hz. Let's just use a R = 100Ω

$$f_{cutoff} = \frac{1}{2\pi RC} - this\ means\ C = \frac{1}{2\pi f_{cutoff} R} = \frac{1}{2\pi(10)(100)} = 159\mu F$$

7. For the circuit below, what is the magnitude of the gain, $\left|\frac{v_o}{v_{in}}\right|$, at 60 Hz?



$$\frac{v_0(t)}{v_s(t)} = \frac{1}{1+\frac{1}{j2\pi fRC}} = \frac{1}{1+\frac{1}{j2\pi(60)(5).0015F}} = 0.889 + j0.314 = 0.943\angle 19.5°$$

Because this problem asks for the magnitude, the answer is 0.942. This means the filter is passing 94.2% the signal, so 60 Hz is in the filter's pass band.