ECE 215 Spring 2025

UNITED STATES
AIR FORCE
ACADEMY

Objective 2.3

I can model capacitors and inductors as complex resistors and use circuit analysis tools (such as voltage dividers, Ohm's Law, and the power equation) to calculate voltage and power in AC circuits containing these components.

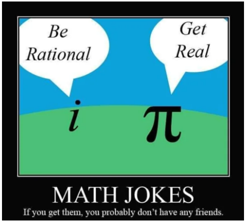
Complex Numbers

3 Forms

- Rectangular Form: $Z = \text{Re} + j(\text{Im}) = A\cos\phi + jA\sin\phi$
- Polar Form: $Z = Ae^{j\phi}$
- Phasor Form: $Z = A\underline{/\phi}$

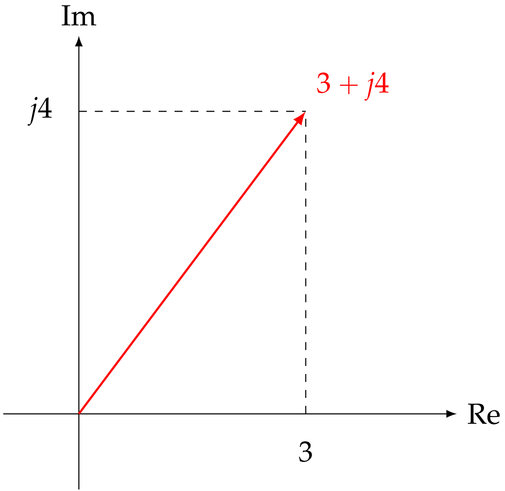
Convert using Euler's Identity: $Ae^{j\phi} = A\cos(\phi) + jA\sin(\phi)$

- Magnitude: $|Z| = A = \sqrt{\text{Re}^2 + \text{Im}^2}$
- Phase: $\phi = \tan^{-1}\left(\frac{\text{Im}}{\text{Re}}\right)$



Complex Plane

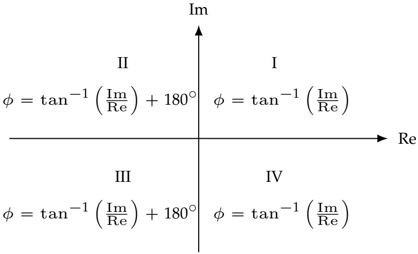
Write the polar and phasor forms of the vector below.



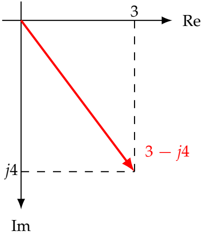
Quadrants

When converting to polar/phasor form, the phase angle depends on which quadrant the complex number lies in.

1. First Quadrant (I): no change
2. Second Quadrant (II): add 180°
3. Third Quadrant (III): add 180°
4. Fourth Quadrant (IV): no change



What is ϕ for this vector?



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How Do I Use My Calculator?

- Use complex mode for calculations
- Convert between rectangular and polar forms

Specifying the Calculation Mode

When you want to perform this type of operation:	Perform this key operation:
General calculations	[MODE] [1] (COMP)
Complex number calculations	[MODE] [2] (CMPLX)

Configuring the Calculator Setup

First perform the following key operation to display the setup menu: [SHIFT] [MODE] (SETUP). Next, use ▼ and ▲ and the number keys to configure the settings you want.

▼ [3] CMPLX [1] a+bi ; [2] $r\angle\theta$ Specifies either rectangular coordinates ($a+bi$) or polar coordinates ($r\angle\theta$) for EQN Mode solutions.

Complex Number Calculations (CMPLX)

To perform complex number calculations, first press [MODE] [2] (CMPLX) to enter the CMPLX Mode. You can use either rectangular coordinates ($a+bi$) or polar coordinates ($r\angle\theta$) to input complex numbers. Complex number calculation results are displayed in accordance with the complex number format setting on the setup menu.

$(2+6i) \div (2i) = 3 - i$ (Complex number format: $a+bi$)
[(] 2 [+] 6 [ENG] (i) [)] [÷] [(] 2 [ENG] (i) [)] [=] **3–i**

$2\angle 45 = \sqrt{2} + \sqrt{2}\,i$ MATH Deg (Complex number format: $a+bi$)
2 [SHIFT] [(−)] ($\angle$) 45 [=] $\sqrt{2}+\sqrt{2}\,i$

$\sqrt{2} + \sqrt{2}\,i = 2\angle 45$ MATH Deg (Complex number format: $r\angle\theta$)
[√] 2 [▶] [+] [√] 2 [▶] [ENG] (i) [=] **2∠45**

Note: • If you are planning to perform input and display of the calculation result in polar coordinate format, specify the angle unit before starting the calculation. • The θ value of the calculation result is displayed in the range of $-180° < \theta \leq 180°$. • Display of the calculation result while Linear Display is selected will show a and bi (or r and θ) on separate lines.

CMPLX Mode Calculation Examples

$(1-i)^{-1} = \frac{1}{2} + \frac{1}{2}i$ MATH (Complex number format: $a+bi$)
[(] 1 [−] [ENG] (i) [)] [x^{-1}] [=] $\frac{1}{2}+\frac{1}{2}i$

$(1+i)^2 + (1-i)^2 = 0$ MATH
[(] 1 [+] [ENG] (i) [)] [x^2] [+] [(] 1 [−] [ENG] (i) [)] [x^2] [=] **0**

To obtain the conjugate complex number of $2+3i$ (Complex number format: $a+bi$)
[SHIFT] [2] (CMPLX) [2] (Conjg) 2 [+] 3 [ENG] (i) [)] [=] **2–3i**

To obtain the absolute value and argument of $1+i$ MATH Deg
Absolute Value: [SHIFT] [hyp] (Abs) 1 [+] [ENG] (i) [=] $\sqrt{2}$
Argument: [SHIFT] [2] (CMPLX) [1] (arg) 1 [+] [ENG] (i) [)] [=] **45**

Conversion Practice

- Convert to rectangular:
  - $10\angle -10^\circ$
  - $5\angle 160^\circ$
  - $7\angle 50^\circ$

- Convert to phasor:
  - $-10 + j6$
  - $5 - j6$
  - $-17 - j4$

Impedance

- Basically frequency-dependent resistance
- Units are ohms (Ω)
- Represent with variable Z
- Recall: $\omega = 2\pi f$

Circuit Components:

- Resistor
  - Linear ($V = IR$) for all time
  - Frequency independent
  - Impedance: $Z_R = R$
- Capacitor
  - $i(t) = C\frac{dv}{dt}$
  - Resists sudden changes in voltage
  - Impedance: $Z_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$
- Inductor
  - $v(t) = L\frac{di}{dt}$
  - Resists sudden changes in current
  - Impedance: $Z_L = j\omega L$

Impedance Practice

Convert the following values to impedances:

- $C = 10\mu\text{F}, f = 200\text{Hz}$
- $L = 20m\text{H}, f = 20\text{Hz}$
- $R = 15\Omega, f = 100\text{Hz}$

RLC Circuit Analysis

With impedance values, we can use all of the circuit analysis tools we've already learned!

- Ohm's Law
- Series/parallel equivalent resistances
- Voltage divider (when in series)
- Current divider (when in parallel)
- KVL and KCL

RLC Circuit Example

Find Z_{eq} and V_C given $v_s(t) = 14.14\cos\left(2\pi * 1.275k * t\right)$V.