Practice Problems

1. What is aliasing and how can it be prevented?

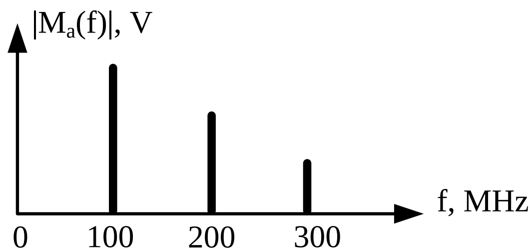
Aliasing is distortion caused by a low sampling rate. It can be prevented by selecting a sampling frequency, f_s, above the Nyquist Rate, which is $2*f_{high}$. It can also be prevented by pre-filtering a signal prior to the analog-to-digital process. The cutoff frequency for the low-pass filter must be less than or equal to $\frac{f_s}{2}$.

2. What are the three steps for converting an analog signal into digital?

   1.) Sampling
   2.) Quantizing
   3.) Encoding

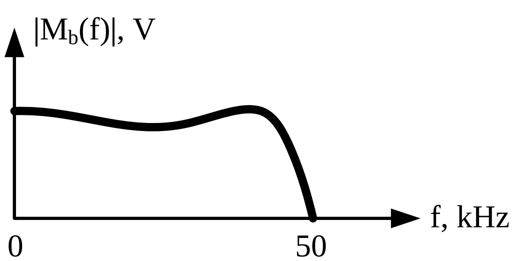
3. Given the following spectra, determine the minimum sampling frequency.

a.



$f_s = 2*f_{high} = 2*300\text{MHz} = 600\text{MHz}$

b.



$f_s = 2*f_{high} = 2*50\text{kHz} = 100\text{kHz}$

4. Find the number of levels and resolution for an 8-bit ADC with V_{max} = 6 V and V_{min} = -4V.

The number of levels is related to number of bits by: levels = 2^b

Substituting in 8 for b, means the number of levels = 2^8 = 256 levels

Resolution is: $\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{6V - (-4V)}{256\ levels} = 39.0625\ {}^{mV}/_{level}$

5. Given a cosine input $v_{in}(t) = 1 + 3cos(360° \ 50kt)V$, answer the following questions:

   a. What is the Nyquist sampling frequency for this input signal?

   $f_s = 2*f_{high} = 2*50kHz = 100kHz$

   b. Which of these sampling frequencies would you use: 75 kHz, 90 kHz, or 120 kHz?

   You must use the 120 kHz sampling frequency. The other two options (75kHz and 90kHz) would result in aliasing because they are below Nyquist.

   c. What are the minimum and maximum values of the input signal?

V_{min}=1+3*min(cos). By definition, min(cos) = -1. Therefore V_{min} = 1+3*(-1) = -2V

V_{max}=1+3*max(cos). By definition, max(cos) = 1. Therefore, V_{max} = 1+3*(1) = 4V

   d. Which of the following ADCs would work with this signal?

- Vmax = 5 V and Vmin = -3 V – The value for V_{min} for the ADC must be less than or equal to the value of V_{min} of the signal. This is calculated above as -2V. This ADC will accept a value of -3V. The value for V_{max} for the ADC must be greater than or equal to the value of V_{max} of the signal. This is calculated above as 4V. this ADC will accept a value of 5V.
- Vmax = 5 V and Vmin = -1 V
- Vmax = 3 V and Vmin = -3 V

   e. Using the V_{max} and V_{min} from part d, how many bits would be required to achieve a resolution of 600 mV or better?

We need to use the resolution equation and manipulate it so that it can be used to solve for b.

$$\Delta V = \frac{V_{max} - V_{min}}{2^b};\ 2^b = \frac{V_{max} - V_{min}}{\Delta V} = \frac{5V - (-3V)}{600mV/level} = 13.333\ levels$$

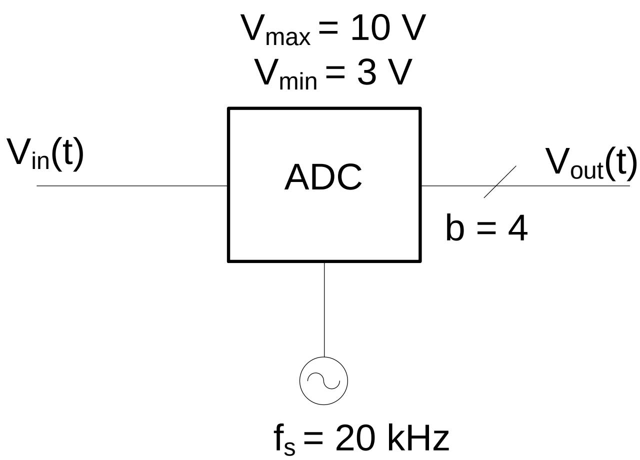
Now, we need to realize we need 14 levels to meet the required resolution. Next, we need to determine the number of bits needed to attain the desired number of levels. The easiest way to do this is to use the "guess" method. 2^1 = 2 levels, meaning an ADC with 1 bit has 2 levels. We can continue in this manner until we find the number of bits that will allow us to meet or exceed 14 levels. 2^2 =4 levels; 2^3 = 8 levels; 2^4 = 16 levels. This means we need 4 bits to meet our resolution requirement for this ADC.

   f. Given the number of bits calculated in part e, what is the actual resolution of the ADC?

We just need to solve the resolution equation using the 4 bits we calculated above.

$$\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{5V - (-3V)}{16} = 500\ {}^{mV}/_{level}$$

6. An input signal, $v_{in}(t) = 6 + 5cos(360°\ 6kt) + 3cos(360°\ 15kt)V$, is to be digitized using the ADC below. Note: the 15 kHz portion of the signal is mainly noise and does not carry any useful information. Is this a good ADC for this signal? Why or why not? If not, what could you do to make this ADC work with this signal?

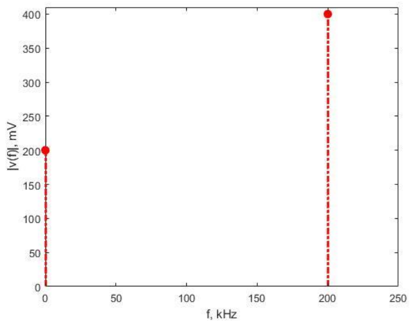


1. Check for aliasing: $f_{s,min} = 2*f_{high} = 2*15kHz = 30kHz > 20kHz = f_{s,ADC}$
    a. Signal will alias. However, we were told the 15kHz signal is noise so we could pass the signal through a LPF prior to the ADC. The LPF $f_{c/o}$ would need to be greater than 6kHz but less than 10kHz to remove the noise. Then the ADC sample frequency would work
2. Check for clipping:
    a. V_{min} = 6 + 5*min(cos) + 3*min(cos) = 6+5*(-1)+3*(-1) = -2V – min clipping will occur
    b. V_{max} = 6 + 5*max(cos) + 3*max(cos) = 6+5*(1)+3*(1) = 14V – max clipping will occur
    c. If 15 kHz frequency is filtered, as recommended in Step 1
        i. V_{min} = 6 + 5*min(cos) = 6+5*(-1) = 1V – min clipping will occur
        ii. V_{max} = 6 + 5*max(cos) = 6+5*(1) = 11V – max clipping will occur
    d. For either condition reviewed (with and without a LPF), clipping will occur. To fix this problem, the signal would need to be run through a transducer interface. When considering the condition set in part (c), the transducer interface requirements would be K= 0.7 and B = 2.3V to make the signal's limits match those of the ADC.

7. You want to convert v(t) to binary numbers to be stored. Your ADC has a Vmax=0.5V and a Vmin=-0.5V and a 3-bit output.

$$v(t) = 200 + 400\cos(360 * 200kt)\,mV$$

Draw the amplitude spectrum of v(t).



a. How would you prepare the signal? Design the transducer interface.

Min: (-200mV)*K + B = -0.5V

Max: (600mV)*K +B = 0.5

Solving gives: K = 1.25, B = -250mV

$V_{out} = 1.25v_{in} - .250V$

b. What sampling rate would you choose?

$f_s = 2*f_{high} = 2*200kHz = 400$ kHz

c. What is the resolution?

$$\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{0.5V - (-0.5V)}{8} = 125\ {}^{mV}/_{level}$$

d. What is the encoded output if the input is 315mV?

$V_{out} = 1.25*0.315 - 0.25 = 0.14375V$

$$EL = \frac{V_{in} - V_{min}}{\Delta V} = \frac{0.14375V - (-0.5V)}{125mV/level} = 5.15\ level, QL = 5\ level$$

2^2	2^1	2^0
4	2	1
1	0	1

8. What is the resolution of a 10-bit ADC with Vmax = 5V and Vmin = -3 V?

$$\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{5V - (-3V)}{1024} = 7.8125\ {}^{mV}/_{level}$$

9. Given the following signal, create an ADC to capture it with 5 bit ADC and determine its resolution.
   $V(t) = \cos(360^0 * 8k * t) + 2 * \cos(360^0 * 15k * t)$ V

V_{min} = 1*min(cos) +2*min(cos) = 1*(-1) + 2*(-1) = -3V

V_{max} = 1*max(cos) +2*max(cos) = 1*(1) + 2*(1) = 3V

$f_s = 2 * f_{high}$ = 2*15kHz = 30kHz

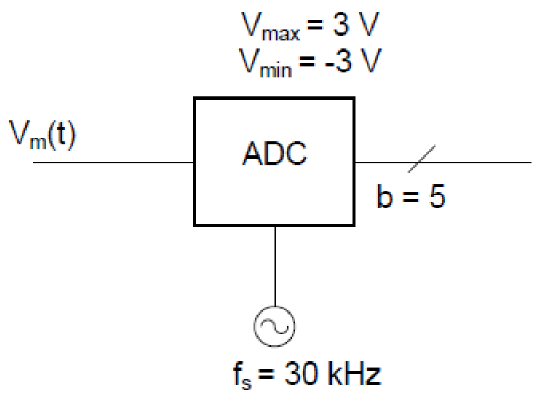
$$\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{3V - (-3V)}{32} = 187.5\ {}^{mV}/_{level}$$

10. Using this ADC encode 1.62 V

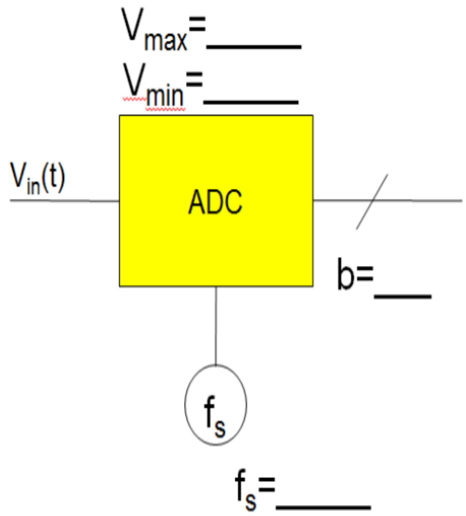
$$EL = \frac{V_{in} - V_{min}}{\Delta V} = \frac{1.62V - (-3V)}{\frac{187.5mV}{level}} = 24.64\ level,$$

$$QL = 24\ level$$

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
1	1	0	0	0



11. Given $v_{in}(t) = 4+4\cos(360^0 \times 2kt)$V, and max QE = 200 mV, find the unknowns in the figure below



V_{max} = 4+4*max(cos) = 8V

V_{min} = 4+4*min(cos) = 0V

f_s = 2*2k = 4kHz

$$\Delta V = \frac{V_{max} - V_{min}}{2^b};\ 2^b = \frac{V_{max} - V_{min}}{\Delta V} = \frac{8V - (0V)}{200mV/level} = 40\ levels$$

b = 6

If a value of 1.6 V is measured, what would be the QE and the encoded value?

$$\Delta V = \frac{V_{max} - V_{min}}{2^b} = \frac{8V - (0V)}{64} = 125\ {}^{mV}/_{level}$$

$$EL = \frac{V_{in} - V_{min}}{\Delta V} = \frac{1.6V - (0V)}{\frac{125mV}{level}} = 12.8\ level, QL = 12$$

QE = ΔV*(EL-floor(EL)) = 0.125*(12.8-12) = 100mV

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1
0	0	1	1	0	0